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BEYOND SCHROEDER DIFFUSERS USING ACOUSTIC METASURFACES

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Resumen
Los difusores de sonido son estructuras diseñadas para controlar la reflexión de las ondas acústicas, que se utilizan principalmente en la acústica de salas para mejorar la calidad del sonido. Sin embargo, como se basan principalmente en resonadores de cuarto de longitud de onda, los difusores clásicos dan lugar a estructuras pesadas y gruesas, especialmente cuando se diseñan para bajas frecuencias. Presentamos un enfoque novedoso para diseñar difusores de sonido con materiales de grosor mucho más pequeño que la longitud de onda, basados en el empleo de la resonancia local, es decir, metamateriales. Los "metadifusores" propuestos muestran una difusión de banda ancha que va de 250 Hz a 2 kHz utilizando paneles de 3 cm, 20 veces más delgados que los diseños tradicionales. Además, su rendimiento puede adaptarse a los diseños clásicos de residuos cuadráticos, raíces primitivas y secuencias ternarias. Además, presentamos una revisión de los avances recientes en este tipo de metasuperficies, incluyendo difusores basados en membranas y placas resonantes, y en vórtices holográficos.

Palabras clave: Difusores de sonido; metamateriales; vórtice; membranas; resonadores de Helmholtz.

Abstract
Sound diffusers are structured surfaces designed to control the scattering of acoustic waves, mainly used in room acoustics to improve sound quality. However, as they are mainly based on quarter-wavelength resonators, phase-grating diffusers result in heavy and thick structures. We present a novel approach to design deep-subwavelength sound diffusers based on subwavelength resonating units, i.e., metamaterials. The proposed "metadiffusers" show broadband diffusion ranging from 250 Hz to 2 kHz using panels of 3 cm, 20 times thinner than traditional designs. In addition, their performance can be tailored to classical quadratic residue, primitive root, and ternary sequence designs. In the presentation we will review recent developments in this type of metasurfaces, including membrane and plate diffusers, and novel holographic approaches.

Keywords: Sound diffusers; metamaterials; vortex; membranes; Helmholtz resonators.

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1 Introduction

The use of architected and resonant surfaces to control sound reflections in room acoustics dates back to the late 70's, when arrangements of quarter-wavelength resonators (QWRs), called phase-grating diffusers, were introduced by M. Schröeder to generate diffuse reflections [1]. These acoustic devices have found practical applications in room acoustics and are widely used in most broadcast studios, modern auditoria, music recording, control, and rehearsal rooms [2]. Schröeder diffusers are reflecting screens based on number theory sequences with flat spatial Fourier transform, initially proposed to generate diffuse reflections. The scattering pattern is essentially driven in the far field by the Fourier transform of the spatially dependent reflection coefficient. A lot of different sequences have been proposed, including bipolar, binary [3], ternary and quadriphase [4] or quadratic-residue [2] sequences. Of special interest are sequences whose first component of the spatial Fourier transform is equal to zero. Indeed, the specular reflection vanishes in this situation, as it does in primitive root or index sequence diffusers. However, the performance of these traditional non-specular sound diffusers is limited because this effect only occurs at the designed frequency. The main drawback of Schröeder diffusers is that they result in thick and heavy structures for low frequencies because they are based in QWRs.

Recently, metamaterials were proposed to reduce the thickness of Schröeder diffusers by using Helmholtz resonators instead of quarter-wavelength resonators and slow-sound metasurfaces [5, 6]. This kind of metasurfaces, initially proposed to design broadband and perfect sound absorbers of subwavelength thickness, [7–9] over a broad range of incidence angles [10]. Therefore, these architected materials allow an accurate control of the phase and amplitude of the reflected field, unleashing the full possibilities to tune the spatial Fourier transform to any complex sequence in a broadband region through the optimization of its geometry.

In this work, we present a summary of the recent advances in sound diffusers based on metamaterials and novel metasurfaces with helical geometry to design broadband and non-specular sound diffusing surfaces.

2 Beyond phase-grating diffusers

The acoustic field at a point $\mathbf{r} = \mathbf{r}(x, y, z)$ scattered by the metasurface located at $\mathbf{r}_0 = \mathbf{r}_0(x, y, z = 0)$ can be approximated by the Rayleigh-Sommerfeld integral as

$$p_s(\mathbf{r}) = -i\frac{k}{2\pi}\int_{S_0} \frac{p_0(\mathbf{r_0})R(\mathbf{r_0})\exp(ik\,|\mathbf{r}-\mathbf{r}_0|)}{|\mathbf{r}-\mathbf{r}_0|}dS_0, \tag{1}$$

where $p_0(\mathbf{r}_0)$ is the incident pressure field, $R(\mathbf{r}_0)$ is the spatially-dependent reflection coefficient of the locally-reacting surface S_0, and $k = \omega/c_0$ is the wavenumber in air at an angular frequency ω, and c_0 is the sound speed.

In the far field, and in spherical coordinates, $\mathbf{r} = \mathbf{r}(\phi, \theta, r)$, using the convention $0 < \phi < 2\pi$ for the azimuth and $0 < \theta < \pi$ for the elevation, the distance between any point and the plane of the metasurface is approximated by

$$|\mathbf{r}-\mathbf{r}_0| \approx r. \tag{2}$$

A second-order Taylor expansion yields

$$|\mathbf{r}-\mathbf{r}_0| \approx r - \frac{x}{r}x_0 - \frac{y}{r}y_0 \approx r - \cos\phi\sin\theta x_0 - \sin\phi\sin\theta y_0. \tag{3}$$

Introducing the approximations given by Eq.(2) and (3) in the denominator and in the phase term of the numerator of Eq. (1), respectively, we get the Fraunhofer-Fourier approximation of the scattered field as

$$p_s(\phi,\theta) = -i\frac{k}{2\pi}\frac{\exp(ikr)}{r}\int_{S_0} p_0(x_0,y_0)R(x_0,y_0)\exp(-i(k_x x_0 + k_y y_0))dx_0 dy_0, \tag{4}$$

where the transversal components of the wavevector are given by

$$\begin{aligned} k_x &= k\cos\phi\sin\theta, \\ k_y &= k\sin\phi\sin\theta. \end{aligned} \tag{5}$$

Note the spherical-divergence factor $\exp(ikr)/r$ is usually dropped as it does not contribute to the directivity of the scattering in the azimuthal and elevation planes.

Equation (4) is essentially a two-dimensional spatial Fourier transform of the reflected field and can be calculated efficiently using fast-Fourier transforms. Therefore, the key to design surfaces with tailored far-field scattering $p_s(\phi,\theta)$ is to find a locally-reacting mechanism to control the complex reflection coefficient $R(x_0,y_0)$ along the surface of the structure. In the following, we present the recent advances to design acoustic devices based on locally-reacting surfaces.

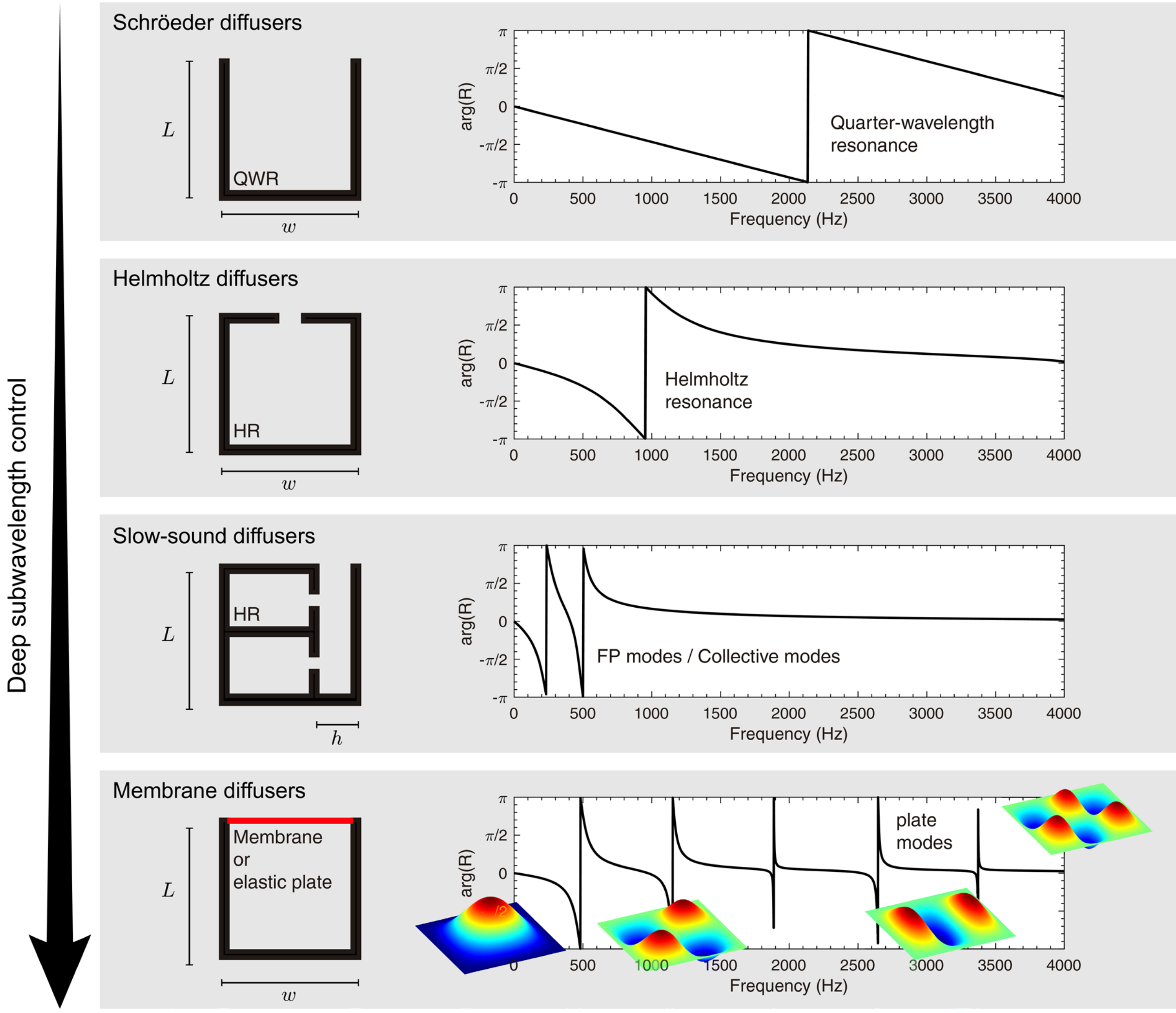

Figure 1. Summary of the recent advances in locally-resonant sound diffusers

The first approach is based on quarter-wavelength resonators (QWR), as illustrated in Fig. (1). These resonators are just perforations in a flat panel, typically in the form of wells of different depths. The reflection coefficient of a given QWR can be tuned by varying the depth of the perforation or well, therefore, it can be tuned to fit a given number-theory sequence. This approach is very convenient because the perforations can be wide, resulting in a good impedance matching of the structure far from the resonance. In this situation, the reflection coefficient of QWRs present a linear phase as a function of the frequency and this enables a broadband response. However, using QWRs the magnitude of the reflection coefficient cannot be tuned as easy as the phase, and, in addition, to obtain a phase up to 2π results in a thick panel, $L \approx c_0/2f_0$, with f_0, the design frequency, see Fig. (1). A convenient solution is to substitute the simple QWRs by other kind of resonators.

Helmholtz resonators (HRs) can be used to downshift the resonance frequency to the deep-subwavelength regime, as shown in Fig. (1). The reflection coefficient can present the whole range of phase values, but at lower frequencies. One might note that this approach results in an impedance mismatch far away from the resonance frequency, therefore, the broadband response of a single resonator is decreased. In addition, irreversible thermoviscous loses appear due to the narrow geometries at the neck. This enables the control of the magnitude of the reflection coefficient, but as there is few geometrical parameters, it become difficult to separately tune the phase and the magnitude.

A more complex geometry can also be used, as e.g., slow-sound metamaterial bricks, as shown in Fig. (1). Using slow-sound metamaterial the number of resonating modes in the low frequency regime can be increased, and as a result, it will be easy to find a complex spatially-dependent reflection coefficient with flat-Fourier transform. Furthermore, the use of narrow slits enables the generation of absorption in the low-frequency regime; so, in addition to the phase, the magnitude of the reflection coefficient can be tuned in a broadband range of frequencies.

Finally, an alternative approach to increase the density of states by introducing addition resonances in the unit cell is to use membrane or plate resonators, as shown in Fig. (1). In this situation, a membrane backed by a cavity show a broad range of resonances corresponding to the plate modes. For thin membranes or plates, these modes can appear at low frequencies, therefore the phase of the reflection coefficient can be tuned with a great degree of freedom in the deep subwavelength regime.

3 Metadiffusers

Figure 1 shows a one-dimensional implementation of a sound diffuser based on deep subwavelength resonators. A slow-sound configuration [5] is used to give to each resonating slot enough degree of freedom to tune the complex reflection coefficient along the surface. In this strategy, we optimized the geometry to maximize the diffusion coefficient. To grant full versatility and obtain high performance, both the phase and the magnitude of the reflection coefficient, i.e., the absorption, must be tuned.

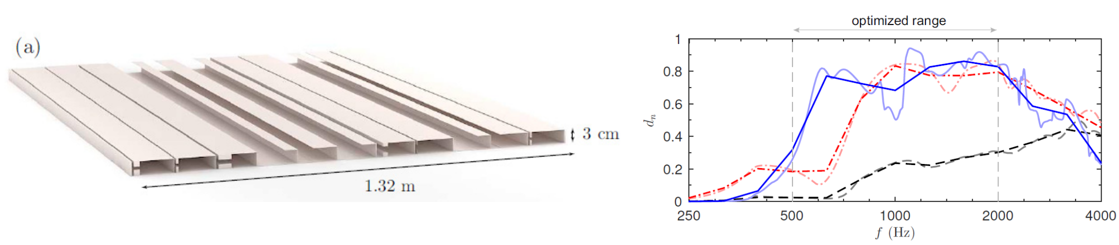

Figure 2. Metadiffuser based on Helmholtz resonators in a slow-sound configuration.

On the one hand, the structure must present a mechanism to control the resonance frequency, resulting in a control of the phase of the reflection coefficient of each slot. This is achieved by tuning the geometry of the Helmholtz resonators. On the other hand, the structure must present a mechanism to control the absorption of each slot. Absorption is produced by activating the thermoviscous loses due to narrow regions. The geometry of this proposal grants a full control over the reflection coefficient of the structure. Therefore, optimization algorithms can find good candidates for broadband sound diffusion using deep subwavelength panels.

4 Membrane metadiffusers

Other strategy is to use membranes or plates instead of quarter-wavelength resonators or Helmholtz resonators. We model the unit cell as a clamped rectangular elastic plate backed by a cavity, as shown in Fig. 3. We follow the model presented by Sun and Jan [11]. For a square clamped plate of side a, the orthogonal modal decomposition of the displacement components of the plate given by $X_m(x)$, $Y_n(y)$ gives

$$\begin{aligned} X_m(x) &= G\left(\frac{\lambda_m x}{a}\right) - \frac{G(\lambda_m)}{H(\lambda_m)} H\left(\frac{\lambda_m x}{a}\right), \\ Y_n(y) &= G\left(\frac{\lambda_n y}{a}\right) - \frac{G(\lambda_n)}{H(\lambda_n)} H\left(\frac{\lambda_n y}{a}\right), \end{aligned} \tag{6}$$

with the functions $G(u)$ and $H(u)$ given by

$$\begin{aligned} G(u) &= \cosh(u) - \cos(u), \\ H(u) &= \sinh(u) - \sin(u), \end{aligned} \tag{7}$$

and where λ_m and λ_n satisfy $\cosh(\lambda)\cos(\lambda) = 1$. Then, we define the following integrals

$$\begin{aligned} I_1 &= \int_0^a X_m(x) \frac{\partial^4 X_m(x)}{\partial x^4} dx, & I_2 &= \int_0^a Y_n^2 dy, \\ I_3 &= \int_0^a X_m(x) \frac{\partial^2 X_m(y)}{\partial x^2} dx, & I_4 &= \int_0^a Y_n(y) \frac{\partial^2 Y_n(y)}{\partial x^2} dy, \\ I_5 &= \int_0^a Y_n(y) \frac{\partial^4 Y_n(y)}{\partial x^4} dy, & I_6 &= \int_0^a X_m(x)^2 dx. \end{aligned} \tag{8}$$

These integrals are numerically integrated using the Simpson's rule. The impedance of the plate is defined as

$$Z = \left[i\omega \int_0^a \int_0^a \left(\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\int_0^a \int_0^a X_m Y_n dx dy}{D(I_1 I_2 + 2 I_3 I_4 + I_5 I_6) - \rho h \omega^2 I_2 I_6} \right) dx dy \right]^{-1}. \tag{9}$$

Finally, the total impedance

$$Z_T = Z_s + i Z_0 \cot(kL), \tag{10}$$

where L is the depth of the cavity. In this case, the resonance frequency can be downshifted deeper in the subwavelength regime. Using plate resonators, as shown in Fig. 3, panels with broadband response can be optimized. However, as the bandwidth of the resonance is narrow, optimized results also present a narrow broadband response. Other drawback is that using plates the absorption coefficient cannot be tuned as easily as using Helmholtz resonators. In addition, practical implementation of membrane and plate resonators usually differs from modelling. We expect that this will result in a performance decrease of optimized panels in realistic situations.

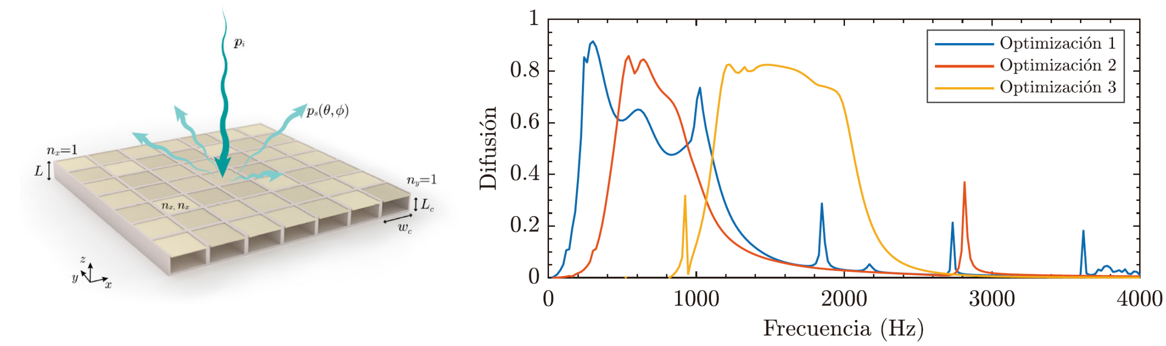

Figure 3. Membrane sound diffuser and corresponding diffusion coefficient for 3 optimized panels.

5 Holographic sound diffusers

Finally, we present a recent approach based on holographic vortices to obtain simultaneously a diffuse response and a no-specular reflection in a broadband response. We start by creating a holographic projection of a virtual vortex into a flat surface. This is achieved by using a back-propagation method using Eq. (1) and setting an acoustic source whose phase rotates an integer number of times.

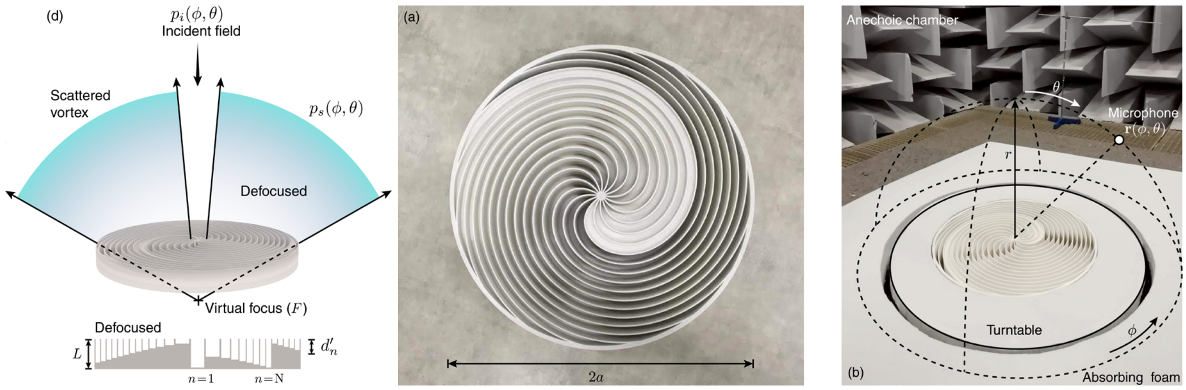

Figure 4. Holographic vortex sound diffusers.

The next step is to create a structure whose reflection coefficient mimics the phase-conjugated recorded field. We achieve this by using a slotted panel whose perforations follow a Fresnel-spiral pattern, as shown in Fig. 4. As each slot behaves as a QWR, the linear phase response allows the holographic pattern to be repeated at multiples of the design frequency. The only difference is that the topological charge of the scattered vortex is increased with frequency.

This results in extraordinary scattering properties as shown theoretically and experimentally in Fig. 5. On the one hand, the specular reflection vanishes because of the phase singularity produced by the vortex, therefore, the correlation scattering parameter reaches values very close to unity in a broadband regime. On the other hand, the scattered waves present a uniform pattern caused by the holographic (de)focusing, resulting in a high value of the normalized diffusion coefficient. Note that diffusion coefficient decreases as the frequency is increased because scattered vortices at high frequencies present a higher topological charge. A high topological charge implies that a broader range of angles surrounding the specular one presents no scattering due to the destructive interference produced by the vortex. However, this results in an excellent correlation scattering coefficient: scattering is no longer specular.

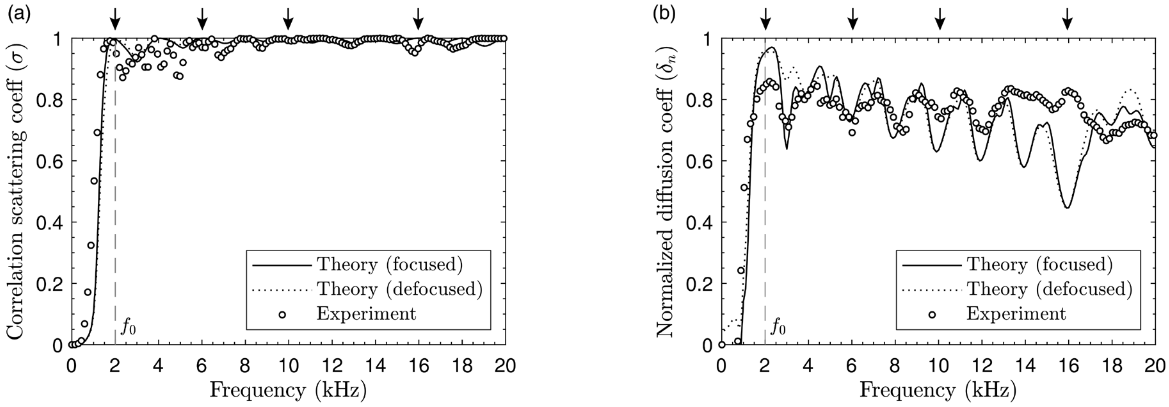

Figure 5. Response of the holographic vortex sound diffusers.

6 Conclusions

Recent advances in metasurfaces have triggered compact designs of sound diffusers. Classical designs were revisited by using locally-resonant acoustic structures, leading to deep-subwavelength structures. In particular, slow-sound metadiffusers or holographic vortex sound diffusers present remarkable compactness and broadband response, enabling its use in modern room acoustic applications to save space, weight, resources, cost and optimize performance.

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